

# $Z_2$ Massive Axions, Domain Walls and Inflation

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## Abstract

We have analyzed a  $U(1)$  model which is broken explicitly to a  $Z_2$  model. The proposal results in generating two stable domain walls, in contrast with the more common  $N_{DW} = 1$  version which is prevalently used to explain axion invisibility for  $UPQ(1)$  model. We have been meticulous to take into account any possible relation with previous studies, even if they apparently belong to different lines of research. Then we have scrutinized the domain wall properties of the model, proposing a rigorous approximate solution which fully satisfies boundary conditions and the static virial theorem simultaneously. Invoking the mentioned approximation, we have been able to obtain an analytical insight about the effect of parameters on the domain wall features, particularly on their surface energy density which is of great importance in cosmological studies when one tries to avoid domain wall energy domination problem. Next, we have mainly focused on the likely inflationary scenarios of the model, including saddle point inflation, again insisting on analytical discussions to be able to follow the role of parameters. We have tried to categorize any inflationary scenario into the known categories to take advantage of the previous detailed studies under the inflationary topic over the decades. We have concluded that any successful inflationary scenario requires large fields definition for the model. Calculations are mainly done analytically and numerical results are used as supportive material.

## 1 Introduction

Recent rigorous observations have provided an unprecedented accuracy that has to be taken into account in any cosmological modeling [1]-[4]. Nowadays, we have enough discriminating data to investigate the practicability of a proposed inflationary scenario precisely [5, 6]. It is widely believed that the recent Planck data favors the simplest inflationary models consisting of a single field slow-roll [6]. Although some inflationary models always remain in the valid domain [7], many of them have been excluded due to incorrect predictions particularly in the density perturbation spectral index on the CMB as well as the power of primordial gravitational waves [2]. This decisive information is at our disposal now, thanks to several experiments and decades of rehearsing on the issue. Simultaneously, we are witnessing some remarkable experiments in particle physics and quantum field theory. No one can doubt that cosmology and quantum field theory are tightly bound and any achievement in one of them must be considered as a clue for the other. There are many attempts to find a QFT motivation as a decisive sign of an acceptable inflationary scenario [8]-[10] and conversely, the capability of a QFT paradigm to include the inflation, is supposed as a supportive sign for the paradigm [11, 12]. On the other hand, after endorsement

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of the Higgs boson existence, the last predicted particle of the standard model [13, 14], there are more attention on the inflationary capability of symmetry breaking scenario [13],[15]-[18]. The measured Higgs mass in the LHC also raised another problem: the Higgs mass and the top quark mass together increase the chance of being in a metastable vacuum for the electroweak theory [19]-[21]. Topologically, a discrete vacuum means domain wall production if the symmetry breaking is **perfect** [22]-[24]. We have known after Zeldovich's 1975 paper [25] that domain walls are drastically in contradiction with the observed cosmic mean energy density unless the domain wall energy density is low enough. Such low energy scales never provide appropriate outline for a successful inflation although the CMB residual dipole anisotropy might be explained using them [26]. The domain wall problem also appears when one tries to solve the strong CP problem by means of introducing a new axion field [27]. Indeed, to explain invisibility of axions due to their weak coupling to matter one could hypothesize more quark species than the usual standard model quarks, or assume two Higgs doublets. The latter case is more appealing in quantum field theory since it offers the modest possible extension to the standard model. Moreover, there are yet unconfirmed reports of observing the footprint of Higgs-like particles in ATLAS and CMS [28], which put the multi-Higgs theories under the spotlight. Assuming a two doublet Higgs scenario, one inevitably encounters even number of domain walls separated by strings. The number of appearing domain walls are two times the number of the generators. If the energy scale of domain walls is high enough such that Domain wall production precedes the inflation, then one has an explanation for not observing such walls, like what happens in magnetic monopoles. Domain walls also could leave no significant remnant in the later stages if they disappear soon enough. There are some known mechanisms for destructing a domain wall which could operate alone or in combination with each other [29]. The most famous one is assuming a metastable domain wall which automatically tends to ruin it [29, 30]. Of course, the decay time could be very long, for example the decay time of electroweak metastable vacuum, in the case of existence, is of the order of the age of the universe [21]. Potentially, unstable domain walls are also among the best candidates for justification of baryogenesis. Amid the other options one can mention destabilizing a domain wall by another defect collision or embarking the symmetron mechanism [31]. There is a very interesting idea that mini black holes could trigger the electroweak vacuum decay in a similar way. On the other hand, one can generalize the natural inflation to include a dynamical modulus in addition to the proposed angular field dynamics. This generalization, respectively, promotes for example  $U_{PQ}(1)$  to the double fields potential in which the  $U(1)$  symmetry is broken to  $Z_n$  discrete symmetry [32]. In this regard, it worth to have an exhaustive analysis of the potential with explicit  $U(R) \rightarrow Z_n$  broken symmetry, to know both the inflationary behavior and possible domain wall properties.

Here, we try a double field potential [33] with two discrete vacua as a toy model of domain wall formation and inflation, trying to avoid rendering numerical results before having an analytical picture. By this, we always keep the track of the model parameters employing appropriate approximations. The assumed potential is very close to the original Higgs potential [34, 35] except that the continuous  $U(1)$  symmetry now is broken into a  $Z_2$  symmetry with two discrete vacua to produce domain walls [36]. To get more familiar with the domain wall properties which potentially could be produced by the proposed model, we first calculate its energy scale for a simple spacial configuration. Results show that entering more parameters into the model and making it more sophisticated provides us with more freedom to control the wall energy scale without decreasing the energy scale of the potential at the origin significantly, in contrast to the Zeldovich's proposal [25]. Next, we discuss the most important possible scenarios in which the potential could accomplish inflation, starting with a complete analytic review of the simple symmetry breaking case according to the recent data. We stick to the most prevailing

method of diagnostic in which the slow-roll parameters play the basic role in the analysis [37, 38]. We also overcome the difficulties of dealing with a double field inflation [39]-[41] by treating the potential as an equivalent single field potential. It soon becomes clear that almost all the scenarios are compatible with the famous hill-top new inflationary models [42]-[44]. Such models of course are categorized among the super-Planckian models with no attainable motivation from known physics, but like other new inflationary models some particular characteristics make them noticeable: They predict very small primordial gravitational waves [2],[45]-[49], much less than what one may hope for detection in a conceivable future. The other point about hill-top models is that there are some techniques to arrange them to work in the supergravity scope [50]-[54].

The outline of the paper is as follows: In the next section, we have a comprehensive review on the previous studies which provide motivations for our survey on descending the  $U(1)$  symmetry to the  $Z_2$ . We focus on fundamental theories skipping applied physics and condensed matter. It is interesting that these different theories are related due to a common characteristic of producing  $Z_2$  domain walls from an original  $U(1)$  symmetry. Then in the third section, we analyze the domain wall characteristics of the proposed potential, where we find a very close approximation as a solution for the domain walls. This approximate solution satisfies the PDE's and static virial theorem simultaneously. Therefore, we invoke this approximation to deal with the other important domain wall characteristics, including the surface energy density and the wall thickness. Afterwards, in the forth section, we propose the potential as the source of inflation, starting from a novel analysis on the ordinary  $U(1)$  symmetry breaking inflation. We try to have a complete survey on all possible scenarios of a successful inflation. We conclude that the saddle point inflation could reduce the scale of the required energy for inflation. This reduction being not such effective to avoid the theory from becoming a super-Planckian. The last section is devoted to the conclusion which contains the most important results of the paper.

## 2 Motivations

Although  $Z_2$  symmetry could not assumed as a formal part of the standard model of particles, it appears frequently for certain reasons. One of these arenas is extending the Higgs sector. In fact, in spite of unnecessary of extra bosons in the standard model, many pioneering theories like Supersymmetry and grand unified theories, demand for extending the Higgs sector. The simplest and the most well-known extended models demand for two Higgs doublet models (2HDMs). 2HDMs also provide one of the best explanations for axion invisibility [27, 51]. Axions are Nambu-Goldstone bosons of Peccei-Quinn  $U(1)$  spontaneous symmetry breaking which originally was invented to solve the strong CP conservation problem [55]. Theoretically, spontaneous breaking of the  $U_{PQ}(1)$  leaves domain wall(s) attaching to a string, [32]. Assuming a multi-Higgs model to deal with the recent unconfirmed record of observing a Higgs-like bump in LHC's last run [28], reinforces the existence of such extensions of the standard model Higgs sector. If the Higgs cousins contain interaction with standard model fermions, which is a very natural postulate, then one can assume  $Z_2$  symmetry and the appropriate fermionic eigen value to avoid Higgs-mediated flavor changing neutral current (FCNC) [56, 57]. Recently, a survey has proposed the cosmological consequences of explicit breaking of  $U_{PQ}(1)$  [32], considering the potential to be

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + \frac{m^2 \sigma^2}{N_{DW}^2} (1 - \cos N_{DW} \theta), \quad (1)$$

where  $\Phi$  is a complex field defined as  $\Phi = |\Phi|e^{i\theta}$ . In their analysis, the responsible term for breaking the  $U(1)$  symmetry demonstrates only the phase field dependence. In our survey we let

the explicit symmetry breaking term to have modulus dependence, too. We therefore assume

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + \frac{m^2 |\Phi|^2}{N_{DW}^2} (1 - \cos N_{DW} \theta). \quad (2)$$

Also for simplicity, we focus on  $N_{DW} = 2$  case since it suffices to inspect the topological domain wall behavior. We are thus led to

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + \frac{m^2 |\Phi|^2}{2} \sin^2 \theta. \quad (3)$$

It is worth mentioning that the above potential form is also a conformally renormalizable extension of natural inflation [58]. Natural inflation, was originally, based on the dynamics of the phase of a complex field whose modulus is stabilized severely. Then the Nambu-Goldstone boson becomes massive thanks to the instanton effect. In the QCD case, instantons break the  $U(1)$  symmetry down to a discrete subgroup to produce the axion-inflaton potential

$$V(\phi) = \Lambda^4 (1 + \cos(\phi/f)). \quad (4)$$

The key prediction of the above form of inflation is the strong gravitational wave remnant. In fact, strong enough to be detected in the recent Planck project. Lacking such approval from observation, one could suppose more completion to the original natural model. Bestowing modulus dependence on the potential is supposed to be one of the first choices. This choice has also been considered recently in [59] by introducing

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + \Lambda^3 (\Phi + \bar{\Phi}). \quad (5)$$

The above potential coincides with KSVZ [27] modification of Pecci-Quinn theory, in which  $N_{DW} = 1$ . In order to promote the above potential to contain two domain walls, which is more desirable in our study, we suggest

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + \Lambda^2 (\Phi + \bar{\Phi})^2. \quad (6)$$

Then assuming  $\Phi = |\Phi| e^{i(\pi/2 - \theta)}$ , one obtains

$$V(\Phi) = \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \sigma^2 \right)^2 + 4\Lambda^2 |\Phi|^2 \sin^2 \theta, \quad (7)$$

which is just (3), with a renaming of the parameters. As it will be introduced later in the paper, our choice of variables for dealing with (3) or (7) is

$$V(R, \Theta) = \frac{\lambda}{4} (R^2 - \sigma^2)^2 + \frac{\epsilon}{2} r^2 \sin^2 \Theta, \quad (8)$$

where

$$R^2 \equiv \Phi^\dagger \Phi = (\phi - i\psi)(\phi + i\psi) = \phi^2 + \psi^2 \quad (9)$$

and

$$\Theta \equiv \tan^{-1} \frac{\psi}{\phi}. \quad (10)$$

From a completely different point of view, in the inflationary paradigm of cosmology, there is a category of potentials, dubbed new inflation, in which the slow roll starts from nearly flat

maximum of the potential where the field(s) is(are) located near the origin. In other words, the slow roll happens to be outward from the origin. These category of inflationary models, survived the tests though they generally suffer super-Planckian parameters. Among new inflationary theories one could mention inverted hybrid inflation [60], which was an attempt to merge new inflation with the hybrid inflation. The potential has the following form

$$V(\phi, \psi) = V_0 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\psi^2\psi^2 - \frac{1}{2}\lambda\phi^2\psi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{4}\lambda_\psi\psi^4. \quad (11)$$

Now if one redefines the parameters as  $\lambda = \lambda_\phi = \lambda_\psi$ ,  $m_\phi^2 = \lambda\sigma^2$ ,  $m_\psi^2 = \epsilon - \lambda\sigma^2$  and  $V_0 = \lambda\sigma^4/4$  then (11) reduces to

$$V(\phi, \psi) = \frac{\lambda}{4}(\phi^2 + \psi^2 - \phi_0^2)^2 + \frac{\epsilon}{2}\psi^2. \quad (12)$$

which is just the Cartesian form of (8). Of course in order to restore the tachyonic instability of the field  $\psi$ , an additional constraint  $\epsilon > \lambda\sigma^2$  is needed, but in our study the latter condition won't be necessary.

### 3 Supersymmetry and explicit breaking of the norm-space $U(1)$ symmetry

In this section, we present a brief motivation for explicit  $U(1)$  symmetry breaking from SUSY. To avoid lengthening the article, we avoid any introductory entrance to supersymmetry. The reader may consult many comprehensive textbooks on the subject. Supersymmetry, if exists at all, must be a broken symmetry. Many attempts have been made to introduce a viable explicit or spontaneous mechanism to explain the supersymmetry breaking. Here, we consider a D-term SSB by adding an additional  $U(1)$  gauge symmetry (Fayet-Iliopoulos mechanism) [61] and derive the resultant potential. We will see that the resulting potential, for the case of two charged scalar fields, demonstrates an explicitly broken  $U(1)$  behavior, when is exhibited in norm-space of the fields. Moreover, since LHC has been obtained no approval evidence for minimal supersymmetric standard model (MSSM) [62], considering additional fields to the supersymmetry sounds as a next logical step. One of the elegant properties of supersymmetry is the automatic appearance of a scalar potential through F and D auxiliary fields which are originally invented to balance the off-shell bosonic and fermionic degrees of freedom [63, 64]:

$$V = V_D + V_F = \frac{1}{2} \sum_a |D^a|^2 + \sum_i |F_i|^2 \quad (13)$$

Supersymmetry requires the vacuume expectation value (vev) to vanish. In this regard, non-vanishing (positive) vev is considered as a sign for SSB. In other words, if both D-term and F-term super potential contributions can't be zero coincidently, then the supersymmetry is broken. One way to do this task is assuming an extra  $U(1)$  symmetry and let the potential to involve a linear D-term besides the ordinary terms:

$$V = \kappa D - \frac{1}{2}D^2 - gD \sum_j q_j |A_j|^2, \quad (14)$$

where  $A_j$  represents the fields that acquire charge under new  $U(1)$  symmetry. The first term, known as Fayet-Iliopoulos (FI) term, satisfies both supersymmetry and  $U(1)$  gauge symmetry.

Then supposing the charged scalar field to be massive, one obtains

$$V = \sum_j |m_j|^2 |A_j|^2 + \frac{1}{2} \left( \kappa - g \sum_j q_j |A_j|^2 \right)^2. \quad (15)$$

Obviously, the above potential has a non-zero minimum which demonstrates breaking of supersymmetry. One has to note that despite broken supersymmetry, the  $U(1)$  gauge remains unbroken if  $|m_j|^2 > \kappa g q_j$  for all fields. If  $A_j$  for all  $j$ 's happen to be massive, then they must appear in pairs with apposite charges to respect the gauge symmetry. Of course, there is no obligation for charged scalar fields to be massive since they can be considered massless without losing any bosonic degrees of freedom. In order to get closer to the model considered in this paper, we consider two massive charged scalar fields  $\phi$  and  $\psi$  and recast  $\lambda = 2g^2$  and  $\phi_0^2 = k/g$  while the charges are normalized to  $\pm 1$ . We obtain

$$V = \frac{\lambda}{4} (|\phi|^2 - |\psi|^2 - \phi_0^2)^2 + m_\phi^2 |\phi|^2 + m_\psi^2 |\psi|^2. \quad (16)$$

Let us redefine the potential in polar coordinates by setting  $|\phi| = R \cos \Theta$  and  $|\psi| = R \sin \Theta$ , then we have

$$V = \frac{\lambda}{4} (R^2 \cos 2\Theta - \phi_0^2)^2 + m_\phi^2 R^2 \cos^2 \Theta + m_\psi^2 R^2 \sin^2 \Theta. \quad (17)$$

Taking derivative with respect to angular coordinate yields

$$\frac{\partial V}{\partial \Theta} = R^2 \sin 2\Theta [\lambda (\phi_0^2 - R^2 \cos 2\Theta) + m_\psi^2 - m_\phi^2] \quad (18)$$

Then two groups of solutions will be gained for  $\frac{\partial V}{\partial \Theta} = 0$ .  $\Theta = k\pi/2$  and  $\Theta = \frac{1}{2} \cos^{-1} \left( \frac{m_\psi^2 - m_\phi^2}{\lambda R^2} + \frac{\phi_0^2}{R^2} \right)$ .

The former is always available but the latter needs an elaborated fine tuning of parameters, particularly, when the  $R$  field is located near the origin initially, like what happens in a normal new inflationary scenario. Therefore, one can consider four attractor paths down from the origin as the slow-roll path, which are equal pairwise.

$$\Theta = (2k+1)\frac{\pi}{2} \implies V = \frac{\lambda}{4} (R^2 - \phi_0^2)^2 + m_\psi^2 R^2 = \frac{\lambda}{4} \left[ R^2 - (\phi_0^2 - \frac{2}{\lambda} m_\psi^2) \right]^2 + m_\psi^2 (\phi_0^2 - \frac{m_\psi^2}{\lambda}), \quad (19)$$

$$\Theta = (2k)\frac{\pi}{2} \implies V = \frac{\lambda}{4} (R^2 - \phi_0^2)^2 + m_\phi^2 R^2 = \frac{\lambda}{4} \left[ R^2 - (\phi_0^2 - \frac{2}{\lambda} m_\phi^2) \right]^2 + m_\phi^2 (\phi_0^2 - \frac{m_\phi^2}{\lambda}). \quad (20)$$

To protect the  $U(1)$  gauge symmetry for each field one requires  $\phi_0 < \sqrt{2/\lambda} m_j^2$ , where  $j$  stands for the corresponding field. One has to note that the potential exhibits the SSB due to its positive definition, whether the gauge symmetry has been broken or not. In the case of  $U(1)$  gauge symmetry breaking, the potential develops one-dimensional kinks, different in shape for each scalar. If both scalar fields are involved in the gauge symmetry breaking, then four vacua will develop. For  $m_\psi = m_\phi$ , all vacua are degenerate and stable, while for  $m_\psi \neq m_\phi$ , the two vacua which are related to the more massive field  $vev_{(metastable)} = m_{(>)} (\phi_0^2 - \frac{m_{(>)}}{\lambda})$ , become metastable and ultimately decay to the two stable vacua  $vev_{(stable)} = m_{(<)} (\phi_0^2 - \frac{m_{(<)}}{\lambda})$ . Since the decay rate decreases exponentially by decreasing the difference of vev's, it is possible to consider the situations in which the metastable lifetime exceeds the age of the universe. Here, some more elaboration is in order. If (16) is assumed for supersymmetric masses set to zero then one obtains

$$V = \frac{\lambda}{4} (|\phi|^2 - |\psi|^2 - \phi_0^2)^2 = \frac{\lambda}{4} (|\phi|^2 + |\psi|^2 - \phi_0^2)^2 + \lambda (\phi_0^2 - |\phi|^2) |\psi|^2, \quad (21)$$

which means that in the space of the charged scalar field norms, the potential shows a  $U(1)$  symmetry, explicitly broken by an effective mass of one of the fields.

## 4 A Two Fields Potential: Domain Wall Analysis

To begin, we propose the simplest asymmetric scenario in a two-fields potential in which we require that  $V(\Phi, \Psi)$  satisfies the following constraint:

$$\frac{\partial V(\Phi, \Psi)}{\partial \Psi} - \frac{\partial V(\Phi, \Psi)}{\partial \Phi} = \epsilon. \quad (22)$$

For  $\epsilon = 0$ , we know that  $V(\Phi, \Psi) = f(\phi + \psi)$  is a solution of (22) for any arbitrary function  $f(x)$ . This ensures  $U(1)$  symmetry if we choose  $\Phi = \frac{\phi^2}{2}$  and  $\Psi = \frac{\psi^2}{2}$ , and consequently  $\epsilon \neq 0$  breaks this symmetry. The above equation has the general solution of the form

$$V(\Phi, \Psi) = f(\Phi + \Psi) + \epsilon \Psi, \quad (23)$$

or

$$V(\Phi, \Psi) = f(\Phi + \Psi) - \epsilon \Phi, \quad (24)$$

where we consider positive definition for  $\epsilon$  throughout the paper. To achieve a more familiar potential form let us recast the fields into  $\Phi = \frac{\phi^2}{2}$  and  $\Psi = \frac{\psi^2}{2}$  and also let the function  $f(\phi, \psi)$  to have an ordinary symmetry breaking appearance. Then the potential (23) could be written as

$$V(\phi, \psi) = \frac{\lambda}{4} (\phi^2 + \psi^2 - \phi_0^2)^2 \pm \frac{\epsilon}{2} \psi^2. \quad (25)$$

The potential (25) shows a full circular symmetry in the first parenthesis resembling the Higgs potential [34, 65]. In the inflationary context this is a self consistent version of a particular hill-top model [47]. In fact, without the last term in (25), the circular freedom in the vacuum corresponds to the massless Goldstone boson [35] but for the case under consideration, the vacuum is not a continuous minimum and as we will see, the circular field acquires mass as well as its own roll down mechanism which has to be considered in slow roll assumption and could bring about different consequences. From now on, we will discuss the symmetry breaking term with the plus sign in (25), but a brief argument about this choice is in order. Suppose we choose the plus sign in (25), Then by the following redefinition

$$\phi'_0 \equiv \left( \frac{\epsilon}{\lambda} - \phi_0^2 \right)^{1/2} \quad (26)$$

and interchanging the variables ( $\phi \rightleftharpoons \psi$ ), one obtains

$$V(\phi, \psi) = V_0 + \frac{\lambda}{4} (\phi^2 + \psi^2 - \phi_0'^2)^2 - \frac{\epsilon}{2} \psi^2, \quad (27)$$

where

$$V_0 \equiv \frac{\epsilon}{4} \left( 2\phi_0'^2 - \frac{\epsilon}{\lambda} \right). \quad (28)$$

So the plus or minus choice for epsilon coefficient is trivial up to a constant. Although breaking the  $U(1)$  symmetry down to  $Z_2$  is very common in condense matter and superconductivity [66], it has received less attention in fundamental theories. In order to have a better perspective about

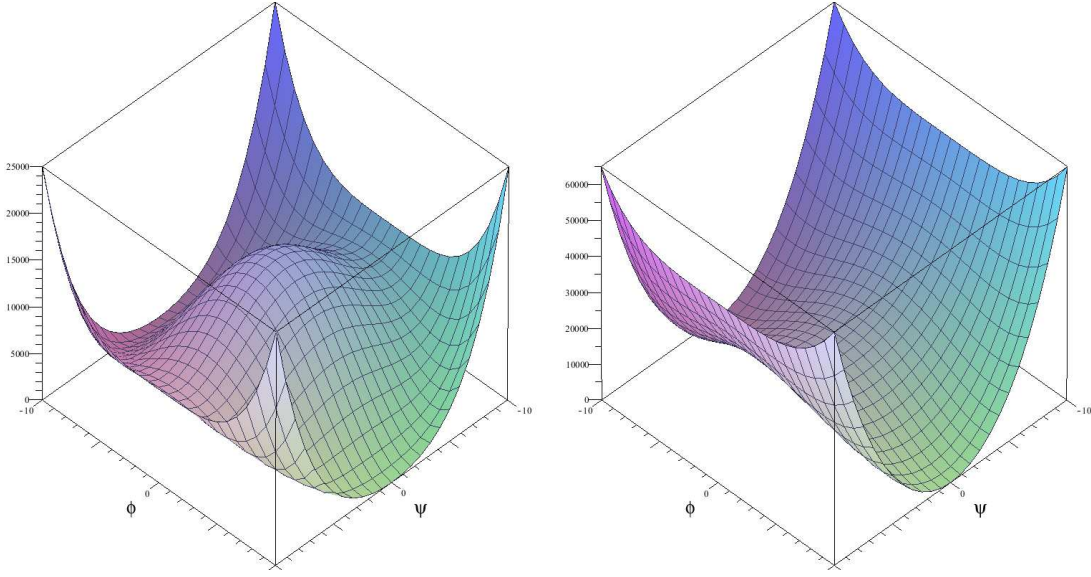


Figure 1: These height plots indicate the potential shape for  $\lambda\phi_0^2 > \epsilon$  (left) and  $\lambda\phi_0^2 < \epsilon$  (right). The origin appears as local maximum for  $\lambda\phi_0^2 > \epsilon$  and is a saddle point for the other case.

the potential, let us change the field coordinates into polar coordinates by setting  $\varphi = R \cos \Theta$  and  $\psi = R \sin \Theta$ . The potential then becomes

$$V(R, \Theta) = \frac{\lambda}{4}(R^2 - \phi_0^2)^2 + \frac{\epsilon}{2}R^2 \sin^2 \Theta. \quad (29)$$

Now we are able to discuss the behavior of the potential by taking a differentiation with respect to the radial field.

$$\frac{\partial V(R, \Theta)}{\partial R} = \lambda R(R^2 - \phi_0^2) + \epsilon R \sin^2 \Theta. \quad (30)$$

To learn about the extrema, let us find the roots of  $\frac{\partial V}{\partial R} = 0$  ;

$$R_{ext} = \{0, \pm \sqrt{\frac{\lambda\phi_0^2 - \epsilon \sin^2 \Theta}{\lambda}}\} \quad (31)$$

Obviously, there could be up to three roots. Here we are able to categorize the potential as bellow

$$\lambda\phi_0^2 > \epsilon \quad \xrightarrow{3 \text{ roots for all } \Theta's} \quad \text{local maximum at the origin} \quad (32)$$

$$\lambda\phi_0^2 \leq \epsilon \quad \xrightarrow[1 \text{ or 3 roots}]{\text{depending on } \Theta} \quad \text{saddle point at the origin} \quad (33)$$

The stronger the inequality (33), the wider range of  $\Theta$  recognizes the origin as minimum. For  $\lambda\phi_0^2 > \epsilon$ , the two saddle points are located at  $\psi = \pm \sqrt{\phi_0^2 - \frac{\epsilon}{\lambda}}$  on the  $\psi$  axis. As the inequality  $\lambda\phi_0^2 > \epsilon$  becomes weaker the saddle points move toward the origin where finally meet the origin for  $\lambda\phi_0^2 = \epsilon$ . The origin remains a saddle point for  $\lambda\phi_0^2 \leq \epsilon$  (Figure 1). Since the potential exhibits discrete vacua after symmetry breaking, domain wall production seems inevitable. More technically, it is the vacuum manifold  $\mathcal{M}$  that determines the character of possible topological



defects [22] and in our case the zero homotopy group, or zero homotopy set as mathematicians prefer, is not trivial ( $\pi_0(\mathcal{M}) \neq 1$ ) which warns us about the inevitability of domain walls generation through the Kibble mechanism [67, 68]. One expects at least one domain wall per horizon volume if the symmetry breaking is **perfect**. The potential (25) has recently been analyzed due to the capability of generating domain walls with a rich dynamics [33]. There is a tight constraint on the existence of domain walls except for very low surface energy densities [25]. Let us see why this is so. For the very popular toy model of  $Z_2$  discrete symmetry breaking in which the effective potential has the  $\phi^4$  well-known form;

$$V(\phi) = \frac{\lambda}{4} (\phi^2 - \phi_0^2)^2. \quad (34)$$

where  $\phi$  is a real field, the scenario has been already analyzed in many references [22, 23, 29, 69] so it would be adequate to review only the results. For simplicity, we assume a Minkowskian background space-time since it suffices to indicate the major properties. After the symmetry breaking is settled down, as a very simple simulation, one could suppose a planar domain wall placed on the  $xy$  plane at  $z = 0$ . For planar domain walls, everything is independent of  $x$  and  $y$  coordinates and as long as we are interested in the static situation, the time dependency is eliminated. So we get

$$\partial^z \partial_z \phi - \frac{\partial V}{\partial \phi} = 0. \quad (35)$$

The first integral of this equation is

$$\frac{1}{2} \sum_i (\partial_z \phi)^2 - V = C. \quad (36)$$

where the constant of integration  $C$  vanishes when we impose the boundary conditions of vanishing of the potential and the spacial field derivative at the infinity. Then the domain wall solution for the assumed boundary condition is

$$z = \int \frac{d\phi}{\sqrt{2V(\phi)}}, \quad (37)$$

or

$$\phi(z) = \phi_0 \tanh(\sqrt{\lambda/2} \phi_0 z). \quad (38)$$

and for the surface energy density of the domain wall we obtain

$$\sigma = \int T^0{}_0 dz = \frac{4}{3} \lambda^{1/2} \phi_0^3. \quad (39)$$

The thickness of the wall is defined as  $(\lambda/2)^{-1/2} \phi_0^{-1}$ . In an expanding universe, any proper velocity of the walls very soon becomes negligible, which leaves the universe with a non-relativistic network of domain walls, and here the problem arises; According to the Kibble mechanism, domain walls are generally horizon-sized so we can estimate their mass as if we do for a horizon-sized plate i.e.  $m_{wall} \sim \lambda^{1/2} \phi_0^3 H^{-2}$ , so the mass energy density can roughly be approximated as  $\rho_{wall} \sim \phi_0^3 t^{-1}$ . We know that the critical density evolves as  $\rho_{crit} \sim t^{-2}$ . Therefore  $R_{wall}(t) \sim (\phi_0/M_{Pl})^2 \phi_0 t$ . By setting  $\phi_0$  at about the GUT scale, the domain wall energy density reaches the critical density already in the time of wall generation and one expects that in our time this ratio becomes  $R_{wall}(t \sim 10^{18}) \sim 10^{52}$  which shows a catastrophic conflict with reality. In order to compromise between the introduced domain walls and the observations, one needs to decrease

the energy of the possible domain walls to very small values [25]. As we will derive in (55) for the more sophisticated domain walls other parameters involve to determine the wall energy which gives us sort of freedom to prepare the potential to work as inflation. The other remedy is to allow the disappearance of domain walls so early that not only diminish from the density calculations but also not altering the CMB isotropy, considerably. This can be achieved in various ways. For example, one can imagine the potential as an effective potential to demote the wall to an unstable version, by which, one of the vacua will disappear through the biased tunneling effect, or considering some sort of destructing collisions which are generally fatal for a kink stability and for such primordial walls the primordial black holes might be the best candidates [58]. It is worth mentioning that domain walls even at very low energies could cause a residual dipole anisotropy in large scale observations, and such an anisotropy is receiving increasing observational supports [26]. Static double-field domain wall solutions corresponding to (25) satisfy the following two coupled Euler-Lagrange equations

$$\frac{\partial^2 \phi}{\partial z^2} = \lambda \phi (\phi^2 + \psi^2 - \phi_0^2), \quad (40)$$

$$\frac{\partial^2 \psi}{\partial z^2} = \lambda \psi (\phi^2 + \psi^2 - \phi_0^2) + \epsilon \psi. \quad (41)$$

These equations can be merged into

$$\frac{1}{\psi} \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{\phi} \frac{\partial^2 \phi}{\partial z^2} = \epsilon. \quad (42)$$

To find the solution of the above equation with appropriate boundary conditions, we have a guide line;  $\phi(z)$  must be odd with respect to  $z$  coordinate due to its main role in the discrete symmetry breaking process, while for  $\psi(z)$  both sides of the wall in  $z = 0$  has the same characteristics since both vacua lay at  $\psi = 0$ . In other words,  $\psi$  has to be even with respect to the  $z$  coordinate. Moreover (42) is subject to the following boundary conditions

$$\phi(z = 0) = 0, \quad (43)$$

$$\phi(z = \pm\infty) = \pm\phi_0, \quad (44)$$

$$\psi(z = \pm\infty) = 0. \quad (45)$$

The trajectory of transition between the vacua doesn't pass through  $(\phi = 0, \psi = 0)$  if  $\phi_0^2 > \epsilon/\lambda$ , since the origin in this case is a local maximum of the potential. To find the domain wall solution, first, we employ an appropriate ansatz for one field and derive the other field. Then we will check the accuracy of the final solution by comparing it with the numerical solutions. Our estimation about the final form must fulfill the boundary conditions. The best choice would be  $\phi(z) = \phi_0 \tanh \sqrt{\epsilon} z$ , this hyperbolic form which has been inspired by the  $\phi^4$  kink solution, fully satisfies the boundary conditions and indicates the odd characteristic of  $\phi(z)$ . The appearance of  $\sqrt{\epsilon}$  is also reasonable since after two times differentiation it will produce the desired  $\epsilon$  factor while the  $\phi_0$  coefficient cancels out by division. Next, we put this ansatz solution into (42), which after some straightforward calculation one obtains for  $\psi(z)$ ;

$$\psi(z) = C_1 \text{sech } \sqrt{\epsilon} z + C_2 \text{sech } \sqrt{\epsilon} z (\sinh 2\sqrt{\epsilon} z + 2\sqrt{\epsilon} z), \quad (46)$$

where  $C_1$  and  $C_2$  are constants of integration. But the above solution has been separated into an odd term and an even term. So to keep the evenness property of  $\psi(z)$  we require  $C_2 = 0$ . To

fix the solution we have to find the remaining constant  $C_2$ . This can be done by means of the minimum energy theorem and integration, but we utilize the static virial theorem, since both of these two theorems stem from the least action principle, they could be used interchangeably. The static virial theorem has another important consequence of vanishing the tangential pressure for the wall, which we prove before inserting it into our calculation. For a typical multi-field potential the Euler-Lagrange equations have the general form

$$\square\chi_a - \frac{\partial V}{\partial\chi_a} = 0, \quad (47)$$

where  $a = \{1, 2, \dots\}$  enumerates the fields. For a static solution and a planar wall these can be written as

$$\frac{d^2\chi_a}{dx_\perp^2} - \frac{\partial V}{\partial\chi_a} = 0, \quad (48)$$

Here  $x_\perp \rightarrow z$  is chosen to be the coordinate perpendicular to the wall. Multiplying by  $\frac{d\chi_a}{dz}$  one obtains

$$\frac{d\chi_a}{dz} \left[ \frac{d^2\chi_a}{dz^2} - \frac{\partial V}{\partial\chi_a} \right] = \frac{d}{dz} \left[ \frac{1}{2} \left( \frac{d\chi_a}{dz} \right)^2 - V(\chi_a) \right] = 0, \quad (49)$$

$$\frac{1}{2} \sum_a \left( \frac{d\chi_a}{dz} \right)^2 - V = \text{constant}. \quad (50)$$

If we require a true vacuum to have zero energy expectation value then the constant of integration in the above equation should be zero. Note that the derived static one dimensional version of virial theorem must be valid for the proposed guess if we require it to satisfy the Euler-Lagrange equation of motion. Obtaining another relation among the fields first derivative and the potential, one can use it in order to determine  $C_1$  in (46) by requiring

$$\left( \frac{d\phi(z)}{dz} \right)^2 + \left( \frac{d\psi(z)}{dz} \right)^2 = 2V(\phi(z), \psi(z)). \quad (51)$$

Substituting for  $\phi(z)$ ,  $\psi(z)$  and  $V(\phi(z), \psi(z))$  and after some algebraic simplification one finds  $C_1 = \sqrt{\phi_0^2 - 2\epsilon/\lambda}$  which results in

$$\phi(z) = \phi_0 \tanh \sqrt{\epsilon} z, \quad (52)$$

$$\psi(z) = \sqrt{\phi_0^2 - 2\frac{\epsilon}{\lambda}} \operatorname{sech} \sqrt{\epsilon} z. \quad (53)$$

This solution approximates very closely the more accurate numerical solution (Figure 2) and the wall width is  $\epsilon^{-1/2}$ . As the next step one can calculate the energy-momentum of the wall

$$T^\mu{}_\nu = \left[ \left( \frac{\partial\phi}{\partial z} \right)^2 + \left( \frac{\partial\psi}{\partial z} \right)^2 \right] \operatorname{diag}(1, 1, 1, 0) = \frac{\epsilon\phi_0^2}{\cosh^2 \sqrt{\epsilon} z} \left( 1 - \frac{2\epsilon}{\lambda\sigma^2} \tanh^2 \sqrt{\epsilon} z \right), \quad (54)$$

Note that although our domain wall solution (64,65) was approximate, the vanishing of  $T^z{}_z = 0$  is an exact result [70]. If we use this result to find the surface-energy density we find

$$\sigma \equiv \int_{-\infty}^{+\infty} T^0{}_0 dz = \int_{-\infty}^{+\infty} (\partial_z\phi)^2 + (\partial_z\psi)^2 dz = 2\sqrt{\epsilon}\phi_0^2 \left( 1 - \frac{\epsilon}{3\lambda\phi_0^2} \right). \quad (55)$$

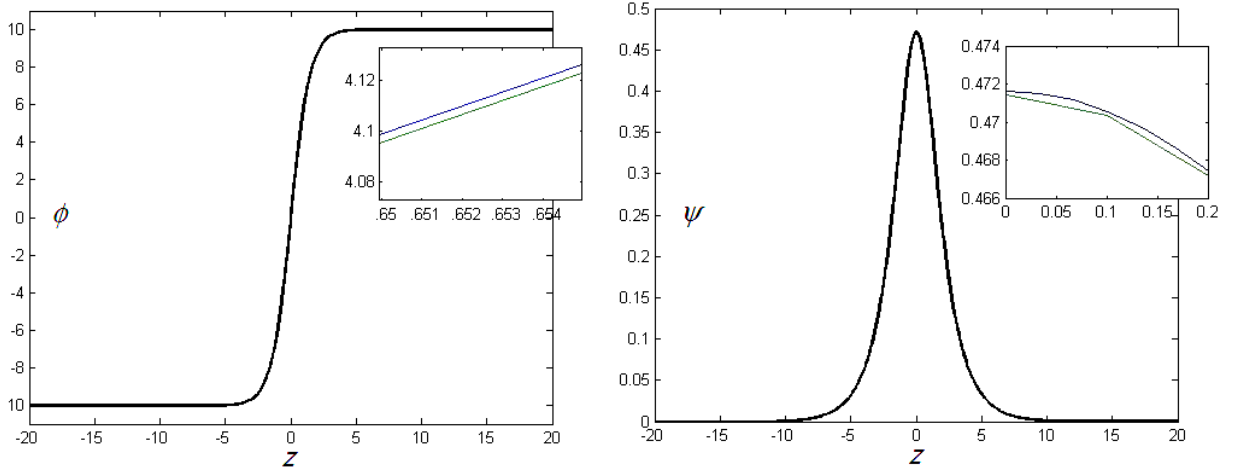


Figure 2: In these figures  $\phi(z)$  and  $\psi(z)$  have been plotted for  $\lambda = 0.009$ ,  $\epsilon = 0.449$  and  $\phi_0 = 10$ , although the parameters have been selected very close to the boundary of the approximation validity ( $\phi_0^2 = \frac{2\epsilon}{\lambda}$ ) still the analytical solutions are very close to numerical solutions. The insets are zoomed to demonstrate the tiny difference.

This result seems interesting since now one can control the effect of  $\phi_0$  by  $\epsilon$  and ultimately for the case of  $\epsilon \ll \lambda\phi_0^2$  one obtains  $\sigma \approx 2\sqrt{\epsilon}\phi_0^2$ , which is independent of  $\lambda$ . This result is noticeable because in contrary to the  $\phi^4$  kink case, now it is possible to decrease the wall energy without decreasing the maximum of the potential, In other words, the peak could be chosen, say at the GUT or even Planck scale, since the wall energy density is low enough to avoid domain wall density domination. To be more clear parameter  $\lambda$  controls the height of the potential through  $\lambda\phi_0/4$  while  $\epsilon$  is responsible for the wall energy density.

The above approximate solution, circumspectly, could be generalized to the below potential

$$V(R, \Theta) = \frac{\lambda}{4}(R^2 - \phi_0^2)^2 + \frac{\epsilon}{2}R^2 \sin^2 \xi \Theta. \quad (56)$$

where  $\xi$  is an arbitrary real number. In order to adapt our postulate to the well-analyzed  $Z_n$  groups, we require  $\xi$  to be integer or half integer. Since other choices of  $\xi$  are related to our line through a field redefinition, the above assumption doesn't demote the level of the generality. Then one has to note that (56) shows explicit breaking of  $U(1)$  symmetry into the  $Z_{2\xi}$  where  $\xi = 1$  restores (29). By this, one obtains  $2\xi$  domain walls, which are equal up to the free Lagrangian. But the picture becomes more contrived if one assumes interactions of  $\phi(R, \Theta)$  and  $\psi(R, \Theta)$  independently, since they acquire different values for each vacuum. Now, let us practice the (64,65) approximation for the new case. Suppose we have a potential with  $\xi = n_0/2$  which provides  $n_0$  domain walls. first we recast the fields into their Cartesian form by  $\phi \equiv R \cos n_0 \Theta$  and  $\psi \equiv R \sin n_0 \Theta$ . Then for the  $n^{th}$  domain wall, the boundary conditions read

$$\phi(z = 0) = 0, \quad (57)$$

$$\phi(z = \pm\infty) = \phi_0 \cos \frac{(n-1)\pi}{n_0}, \quad \phi(z = \mp\infty) = \phi_0 \cos \frac{n\pi}{n_0}, \quad (58)$$

and

$$\psi(z = \pm\infty) = \phi_0 \sin \frac{(n-1)\pi}{n_0}, \quad \psi(z = \mp\infty) = \phi_0 \sin \frac{n\pi}{n_0}, \quad (59)$$

where each line demonstrate two distinct possibilities for each sign. Obviously, the odd and even properties of the fields, which are crucial for the next steps, disappear. In order to restore the parity characteristics in boundary behavior one needs to rotate the fields by angel  $\alpha = (n - \frac{n_0+1}{2})T$ , where  $n$  counts the number of the intended walls when we enumerate them counter clockwise from  $\Theta = 0$  and  $T = \frac{\pi}{n_0}$  is the angular period of the potential.

$$V = \frac{\lambda}{4} (R^2 - \phi_0^2)^2 + \frac{\epsilon}{2} R^2 \sin^2(\Theta' = \Theta + \alpha). \quad (60)$$

Then, for even  $n_0$ , the potential remains the same since the rotation angel is an integer times the period. But for  $n_0$  odd, further elaboration is needed. In the rotated cordinate the boundary conditions are

$$\phi'(z = 0) = 0, \quad (61)$$

$$\phi'(z = \pm\infty) = \pm\phi_0 \sin \frac{\pi}{2n_0}, \quad (62)$$

$$\psi'(z = \pm\infty) = \phi_0 \cos \frac{\pi}{2n_0}, \quad (63)$$

which exhibit the desired parity. Therefore the next steps are eligible and the approximation becomes

$$\phi'(z) = \phi_0 \sin \frac{\pi}{2n_0} \tanh \sqrt{\epsilon} z, \quad (64)$$

and

$$\psi'(z) = \sqrt{\phi_0^2 \sin^2 \frac{\pi}{2n_0} - 2\frac{\epsilon}{\lambda}} \operatorname{sech} \sqrt{\epsilon} z. \quad (65)$$

The validity bound of the above approximation becomes narrower by increasing  $n_0$ , since it requires  $(\phi_0^2 \sin^2 \frac{\pi}{2n_0} - 2\frac{\epsilon}{\lambda}) > 0$ . One can return to the original coordinates by a simple reverse rotation, respectively;

$$\begin{bmatrix} \phi \\ \psi \end{bmatrix} = \begin{bmatrix} \cos \left( \frac{n_0+1}{2} - n \right) \frac{\pi}{n_0} & -\sin \left( \frac{n_0+1}{2} - n \right) \frac{\pi}{n_0} \\ \sin \left( \frac{n_0+1}{2} - n \right) \frac{\pi}{n_0} & \cos \left( \frac{n_0+1}{2} - n \right) \frac{\pi}{n_0} \end{bmatrix} \begin{bmatrix} \phi_0 \sin \frac{\pi}{2n_0} \tanh \sqrt{\epsilon} z \\ \sqrt{\phi_0^2 \sin^2 \frac{\pi}{2n_0} - 2\frac{\epsilon}{\lambda}} \operatorname{sech} \sqrt{\epsilon} z \end{bmatrix} \quad (66)$$

## 5 Inflationary Analysis

The proposed potential in an inflationary perspective should be categorized within the "new inflation" models [71], while the inflation is expected to begin near the maximum where the fields leave the origin. New inflationary scenarios received great welcome because they do not have the common problems of the old inflation [72] in completion of inflation [8, 49]. In fact, old scenarios need bubble collisions but a new inflationary scenario can end with a more realistic process of oscillation around a minimum [43, 44, 71]. Here, we try to scrutinize how (25) works as an inflationary potential, too.

### 5.1 Inflationary Scenario 1 : An Analytical Overview on Simple Symmetry Breaking Inflation

The extreme situation occurs when epsilon is considered as a small perturbation parameter in the original symmetry breaking term and can be ignored for the most of the process, then the potential is readily reduced to the single potential the same as the simple symmetry breaking which has been already explored in some respects [49]. Henceforth, in order to have a measure

for the remaining part of our survey it is convenient to know the inflationary characteristics of the potential when the asymmetric term is ignorably small. To provide more similarity we can rewrite the potential as

$$V(R) = \Lambda \left(1 - \frac{R^2}{\phi_0^2}\right)^2, \quad (67)$$

in which we have used

$$R = (\phi^2 + \psi^2)^{\frac{1}{2}}, \quad (68)$$

$$\Lambda = \frac{\lambda \phi_0^4}{4}. \quad (69)$$

Obviously, this potential is now in the domain of "New Inflation", in which the field rolls away from an unstable equilibrium, here placed at the origin. Even after considering the whole potential, this will remain the main theme of the analysis. We can proceed by making a Taylor expansion keeping the leading terms and ignoring the rest due to  $R^4 < \phi_0^4$  for the roll down path.

$$V(R) \approx \Lambda \left(1 - \left(\frac{R}{\mu}\right)^2\right), \quad (70)$$

where

$$\mu = \frac{\phi_0}{\sqrt{2}}. \quad (71)$$

It is a well-known result that a potential in the form  $V(r) = \Lambda(1 - (\frac{r}{\mu})^\alpha)$  works properly as an inflationary potential for  $\alpha = 2$  if it is LFM (Large Field Model) i.e.  $r \geq M_{Pl}$  [71]. To be more precautionous and to have a measure for the coming procedure, let us see the case more closely. The slow roll parameters [73] are

$$\varepsilon = \frac{8M_{Pl}^2 R^2}{(\phi_0^2 - R^2)^2} \quad (72)$$

and

$$\eta = \frac{4M_{Pl}^2(3R^2 - \phi_0^2)}{(\phi_0^2 - R^2)^2}. \quad (73)$$

To estimate the field value at the end of inflation we require  $\epsilon_{end} \sim 1$  [73], then the appropriate solution would be

$$R_{end} = \sqrt{4M_{Pl}^2 + \phi_0^2 - 2\sqrt{4M_{Pl}^4 + 2M_{Pl}^2\phi_0^2}}. \quad (74)$$

Assuming  $\alpha \equiv \frac{M_{Pl}}{\phi_0} < 1$  and utilizing the Taylor expansion in favor of the leading terms we can write (74) as

$$R_{end} \simeq \phi_0 \sqrt{1 - 2\sqrt{2}\alpha + 4\alpha^2 - 2\sqrt{2}\alpha^3}. \quad (75)$$

One can recognize that the end of inflation happens close to the true minimum. Having an estimate for  $R_{end}$ , we are able to obtain the e-folding interval between the time that cosmological scales leave the horizon and the end of inflation [73]

$$N \approx \int_{R_{end}}^{R_*} \frac{V}{V_{,R}} dR = \left(\frac{1}{8}R^2 - \frac{1}{4\alpha^2} \ln R\right)_{R_{end}}^{R_*} = -\frac{1}{4\alpha^2} \ln \left(\frac{R_*}{R_{end}}\right) + \frac{1}{8} \left(\frac{1}{R_*^2} - \frac{1}{R_{end}^2}\right), \quad (76)$$

where we set  $M_{Pl} = 1$  for simplicity. Substituting (75) in (76) and making some straightforward approximation again, one obtains

$$N \approx -\frac{1}{8\alpha^2} \left[ \left( \frac{R_*}{R_{end}} - 2 \right) - \left( \frac{R_*}{R_{end}} - 2 \right)^2 \right] + \frac{1}{8} \left( \frac{1}{R_*^2} - \frac{1}{R_{end}^2} \right) = \frac{1}{8\alpha^2} \left[ (2 - R_*\alpha)^2 - 2(1 - \sqrt{2}\alpha) + R_*^2\alpha^2 \right]. \quad (77)$$

One can solve the above equation with respect to  $r_*$  for the only acceptable solution:

$$R_* = \frac{1 - \sqrt{4N\alpha^2 - \sqrt{2}\alpha}}{\alpha}. \quad (78)$$

So far we supposed  $\alpha < 1$  to validate our approximation and now  $\alpha$  appears in the denominator. One can readily justify that  $R_*$  is a monotonically decreasing function of  $\alpha$ . It implies that lowering the  $\alpha$  raises the field value in which the desired scale leaves the horizon. This statement is reasonable since smaller  $\alpha$  leads to a decrease in the slope of the potential with respect to the field  $R$ . This provides us a straightforward method to find the maximum allowed value for  $\alpha$  in which  $R_*$  coincides with the origin i.e.  $R_* = 0$ , then the answer will be

$$\alpha_{max} = \frac{\sqrt{2} + \sqrt{2 + 16N}}{8N}. \quad (79)$$

Therefore from (79), if one requires  $N > 50$ , it roughly means  $\alpha < 0.0743$ . which is in accord with the previous assumption about smallness of  $\alpha$ . To have an insight, we have to emphasize that this upper bound for  $\alpha$  coincides with ignorable  $R_*$  and undetectably low value for primordial gravitational waves as it will become clear shortly.

Now let us have a look at the most important observational constraints on any inflationary hypothesis; spectral index and tensor to scalar perturbation ratio. For the spectral index we obtain

$$n_s - 1 \approx 2\eta_* - 6\epsilon_* = -\frac{8(3R_*^2 * \phi_0^2)}{(R_*^2 - \phi_0^2)^2} \approx -8\alpha^2(1 + 5\alpha^2 R_*^2) \approx -8\alpha^2 \left[ 1 + 5(1 - \sqrt{4N\alpha^2 - \sqrt{2}\alpha})^2 \right]. \quad (80)$$

This estimation is accurate enough to indicate that for  $\alpha = 0$  one regains the scale-invariant Harrison-Zel'dovich-Peeble's [74] spectrum as expected. Recall that we keep assuming  $M_{Pl} = 1$ . Let us use the fact that tensor to scalar perturbation ratio "r";  $r = 16\epsilon_*$  must be smaller than 0.11 [1]. Combining the definition of "r" with (72) and making some simplification yields

$$r \approx \frac{128R_*^2}{\phi_0^2(\phi_0^2 - R_*^2)^2}. \quad (81)$$

which points to a vanishingly small  $r$  when the horizon exit happens near the origin ( $R_* = 0$ ) as mentioned before. Substituting for  $R_*$  we obtain

$$r \approx 256\kappa^2\alpha^2(\kappa^2 + \frac{1}{2}), \quad (82)$$

where  $\kappa$  is defined as

$$\kappa \equiv \pm \left[ 1 - \sqrt{\alpha(4N\alpha - \sqrt{2})} \right]. \quad (83)$$

Then for the spectral index we have

$$n_s - 1 = -\frac{(1 - 9r_*^4\alpha^4)}{16r_*^2\alpha^4}r = \frac{(9\kappa^4 - 1)r}{16\kappa^2\alpha^2} \quad (84)$$

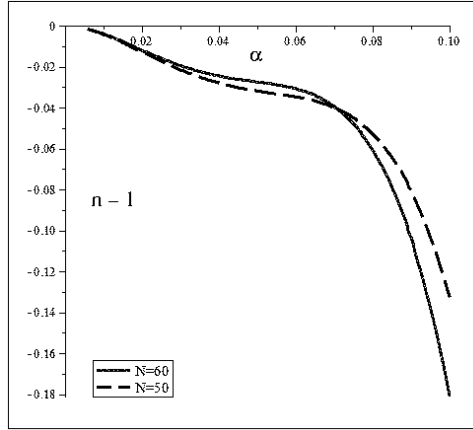


Figure 3: This graph indicates  $n_s - 1$  with versus  $\alpha$  for the range of e-folding ( $N_{min} = 55$  and  $N_{max} = 65$ ). Therefore one can claim that for the model under investigation, spectral index is a monotonically decreasing function of  $\alpha$ .

This estimation is not precise enough yet but helps us to have a better insight. Assuming a lower value for  $\alpha$  means that  $R_*$  moves away from the origin but this can not effect the gravitational wave strength considerably, since (81) can always be approximated as

$$r \approx 128R_*^2\alpha^6. \quad (85)$$

From (80) one can verify that  $n_s - 1$  is a monotonically decreasing function of  $\alpha$  in the allowed range  $\alpha < 1/13.5$  (Figure 3). But we have an accurate observation for the scalar spectral index by Planck TT+lowP 68% CL, [1, 4];

$$0.9593 < n_s < 0.9717. \quad (86)$$

This yields the range of valid  $\alpha$  for a typical e-folding  $N = 55$

$$0.046 < \alpha < 0.071. \quad (87)$$

This means  $14M_{Pl} < \phi_0 < 21.8M_{Pl}$  which exhibits a relatively stringent fine tuned need of the model. On the other hand, the estimated lower limit for  $\alpha$  can help us to determine the maximum of  $R_* \times \alpha$  since from (78) we have

$$R_*\alpha = 1 - \sqrt{4N\alpha^2 - \sqrt{2}\alpha}, \quad (88)$$

which is again a decreasing function of  $\alpha$  in the permitted region  $0.046 < \alpha < 0.071$  such that  $R_* \times \alpha$  remains between 0 and 0.04, approximately. One can estimate the tensor to scalar ratio using (88) in (85) to obtain

$$r \approx 128\alpha^2 \left( 1 - \sqrt{4N\alpha^2 - \sqrt{2}\alpha} \right)^2. \quad (89)$$

This function monotonically decreases with respect to  $\alpha$  in the acceptable range of  $\alpha$ , with the maximum value about  $r = 0.04$  for  $\alpha_{min} = 0.046$ . As one expects, the tensor to scalar ratio vanishes for  $\alpha_{max} = 0.071$  since it requires the initial point to be the fixed point (Figure 4).



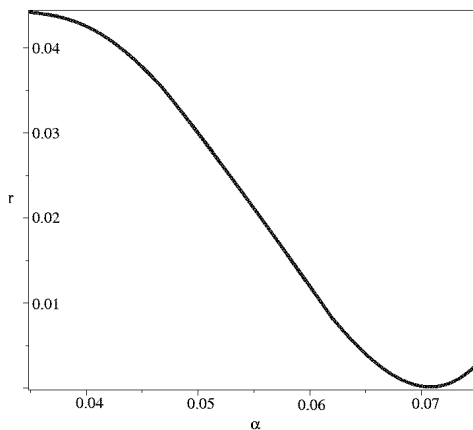


Figure 4: Plotting the approximate result for the tensor to scalar ratio shows that for the target range of  $\alpha$  one can claim  $0 < r < 0.04$ , which is a typical behavior for a hill-top model.

Therefore we learn that the model needs a super-Planckian value for  $\phi_0$  to work properly according to the available data from Planck+WP+BAO [1, 2]. There is another piece of observational information yet to be addressed. Actually the oldest and the most well-known part of it. From the expansion of the power spectra for the curvature perturbations we have [2]

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left( \frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} \frac{dn_s}{d \ln k} \ln(k/k_*) + \dots} \quad (90)$$

Thus, the Planck constraint on  $r$  implies an upper bound on the inflation energy scale

$$\max(V_*) = \frac{3\pi^2 A_{s*}}{2} r_* M_{Pl}^4, \quad (91)$$

which is readily transformed to the more useful form

$$\frac{H_*}{M_{Pl}} < 3.7 \times 10^{-5}. \quad (92)$$

Taking into account the slow-roll paradigm once more, we obtain

$$\sqrt{\frac{V_*}{3}} < 3.7 \times 10^{-5}. \quad (93)$$

which equivalently means

$$\lambda < 3 \times 10^{-13}. \quad (94)$$

## 5.2 Inflationary Scenario 2: The Proposed Potential and Possible Inflationary Slow-rolls

For the potential (25) the dynamics is limited between two extrema. The first is similar to the previous case where the field starts rolling down from the vicinity of the  $\psi = 0$  axis and remains close to it throughout its motion. This is the most probable possibility due to the angular minimum of the potential and this is more or less the only possibility if one takes  $\epsilon > \lambda \phi_0^2$ , because as it was discussed, in this situation, the origin becomes a saddle point and  $\psi = 0$  seems

like an attractor path. But if we assume  $\epsilon < \lambda\phi_0^2$ , then other paths are possible depending on different initial conditions, reminding that the nearer to the  $\psi$  axis the less chance for the path to be chosen, again because of the angular behavior of the potential. But as a possibility, though a very weak one, we consider the ultimate radial path from the origin to one of the side saddle points at  $\psi = \pm\sqrt{\phi_0^2 - \epsilon/\lambda}$  and then a curved orbit toward the true vacuum. Note that the second part of the path could happen independently from the starting point. At the point where these different trajectories meet, some transient oscillations might happen, which are damped by the following inflationary period. Generally, these two extreme paths could bring different expansions as we will discuss. For the moment, let us concentrate on the path from the origin toward the  $\psi = \sqrt{\phi_0^2 - \epsilon/\lambda}$  saddle point. Remember that although this is not an attractor and the chance of following this path is low, still as we emphasized earlier, we consider it as a bound for what could happen. As long as we restrict ourselves to move on the  $\phi = 0$  axis, the effective potential can be simplified as

$$V(\phi = 0, \psi) = \frac{\lambda}{4}(\psi^2 - \phi_0^2)^2 + \frac{1}{2}\epsilon\psi^2, \quad (95)$$

which can be reordered as

$$V(\psi) = \frac{\lambda}{4} \left( \psi^2 - \left( \phi_0^2 - \frac{\epsilon}{\lambda} \right) \right)^2 + \frac{\epsilon}{2} \left( \phi_0^2 - \frac{\epsilon}{2\lambda} \right). \quad (96)$$

It can be recognized now that the potential is similar to (67) shifted by a constant. So we can proceed in a similar manner unless this time we set  $\psi_{end} = \sqrt{\phi_0^2 - \epsilon/\lambda}$  to obtain the amount of e-foldings in this part of the slow-roll path;

$$N \approx \frac{1}{M_{Pl}^2} \int_{\psi_{end}}^{\psi_*} \frac{V}{V, \psi} d\psi \approx \frac{1}{4\alpha^2} \ln\left(\frac{\psi_{end}}{\psi_*}\right) - \frac{1}{8}\psi_{end}^2. \quad (97)$$

Actually, the inflation couldn't reach the saddle point at  $\phi = 0$  and  $\psi = \sqrt{\phi_0^2 - \frac{\epsilon}{\lambda}}$  since it is interrupted at  $\psi^2 \approx (\phi_0^2 - \epsilon/\lambda) + 2M_{Pl}(3M_{Pl} - \sqrt{2}\phi_0)$  due to the growth of the slow-roll parameter  $\eta$ . This is a transitive situation and inflation starts quickly in a new path as we will consider shortly. The whole e-foldings through  $\phi = 0$  trajectory is

$$N \approx \left( \frac{1}{8}\psi^2 - \frac{\lambda\phi_0^4 \ln \psi}{4(\lambda\phi_0^2 - \epsilon)} + \frac{\epsilon \ln(\lambda\phi_0^2 - \lambda\psi^2 - \epsilon)}{4(\lambda\phi_0^2 - \epsilon)} - \frac{\epsilon^2 \ln(\lambda\phi_0^2 - \lambda\psi^2 - \epsilon)}{8\lambda(\lambda\phi_0^2 - \epsilon)} \right)_{\psi_{end}}^{\psi_*}. \quad (98)$$

To simplify the above expression, we invoke two facts; first we know that the pivot scale leaves the horizon soon near the origin ( $\psi_0 \approx 0$ ) and second, the inflation stops essentially before  $\psi = \sqrt{\phi_0^2 - \lambda/\epsilon}$  so the last two terms make no considerable contribution and we finally obtain

$$N \approx -\frac{\lambda\phi_0^4 \ln \psi_*}{4(\lambda\phi_0^2 - \epsilon)}. \quad (99)$$

It is seen that the denominator could intensify the e-foldings by providing a semi-flat trajectory on the  $\phi = 0$  axis. But remember that this path is not likely to happen due to instability. Now we can write

$$\psi_* \approx e^{-4N \left( \frac{\phi_0^2 - \epsilon/\lambda}{\phi_0^4} \right)}. \quad (100)$$

However the effect of varying  $\epsilon/\lambda\phi_0^2$  only slightly changes our picture about  $\psi_* \approx 0$  since

$$\lambda\phi_0^2 > \epsilon > 0 \implies 0.37 M_{Pl} < \psi_* < M_{Pl}. \quad (101)$$

All other features along this path more or less resemble the ordinary symmetry breaking case which was discussed earlier and we will consider this again from a different view. But inflation could happen along a completely different path; starting from the saddle point and ending at the true vacuum. To have an estimation about the selected path due to its low kinetical energy we suppose that the fields remain in the radial minimum throughout their trajectory. Therefore for obtaining the equation of the estimated path first we find the radial minimum of the potential in the polar form (29)

$$\frac{\partial V}{\partial R} = \lambda R(R^2 - \phi_0^2) + \epsilon R \sin^2 \Theta = 0. \quad (102)$$

Ignoring the  $R = 0$  solution, which correspond to the maximum at the origin, for a given angle  $\Theta$  the radial minimum obeys the following equation

$$R_{min}^2(\Theta) = \phi_0^2 - \frac{\epsilon}{\lambda} \sin^2 \Theta. \quad (103)$$

Returning to the original Cartesian form, we have

$$\phi^2 + \psi^2 = \phi_0^2 - \frac{\epsilon}{\lambda} \frac{\psi^2}{\phi^2 + \psi^2}. \quad (104)$$

Setting  $\epsilon = 0$  we recover the  $U(1)$  symmetry as expected. The true vacua also satisfy the above equation. To simplify the analysis, we will focus on the  $\psi > 0, \phi > 0$  quarter and solve (104) for  $\phi^2$  to obtain

$$\phi^2 = \frac{\phi_0^2}{2} - \psi^2 + \frac{\phi_0^2}{2} \sqrt{1 - \frac{4\epsilon\psi^2}{\lambda\phi_0^4}} \quad (105)$$

since we assume the  $\epsilon \ll \lambda\phi_0^2$  regime, expanding the square root and keeping the most important terms we deduce

$$\phi^2 = \phi_0^2 - \psi^2 \left( 1 + \frac{\epsilon}{\lambda\phi_0^2} \right), \quad (106)$$

or equivalently

$$\psi = \sqrt{\frac{\phi_0^2 - \phi^2}{1 + \frac{\epsilon}{\lambda\phi_0^2}}}. \quad (107)$$

Finally the above approximation allows us to recast the potential (25) in the form of a single field potential

$$V(\phi, \psi) = \sqrt{\frac{\phi_0^2 - \phi^2}{1 + \frac{\epsilon}{\lambda\phi_0^2}}} = \frac{\lambda\epsilon\phi_0^2(\phi_0^2 - \phi^2)(3\epsilon + 2\lambda\phi_0^2 - \epsilon\frac{\phi^2}{\phi_0^2})}{(\lambda\phi_0^2 + \epsilon)^2}, \quad (108)$$

while for the supposed slow-roll path  $\phi < \phi_0$  we estimate

$$V(\phi) \approx \frac{\lambda\epsilon\phi_0^4(3\epsilon + 2\lambda\phi_0^2)(1 - \phi^2/\phi_0^2)}{4(\lambda\phi_0^2 + \epsilon)^2}. \quad (109)$$

Then obviously for  $\lambda\phi_0^2 \gg \epsilon$  we obtain

$$V(\phi) \approx \frac{\epsilon\phi_0^4}{2} \left( 1 - \frac{\phi^2}{\phi_0^2} \right). \quad (110)$$

As we expected, the inflation energy scale is determined by  $\epsilon$  and  $\phi_0$  since the height of the barrier depends on them. Under this condition, the potential completely fits to the first new inflationary models called hill-top models [42] with the general shape

$$V(\phi) = \Lambda^4 \left( 1 - \frac{\phi^p}{\mu^p} + \dots \right), \quad (111)$$

if we set  $p = 2$ ,

$$\Lambda^4 = \frac{\epsilon(2\phi_0^2\lambda - \epsilon)}{4\lambda}, \quad (112)$$

and

$$\mu = \phi_0. \quad (113)$$

Then the model predictions are

$$n_s - 1 \approx \frac{-4M_{Pl}^2}{\mu^2} + \frac{3r}{8}, \quad (114)$$

and

$$r \approx \frac{32\phi_*^2 M_{Pl}^2}{\mu^4}, \quad (115)$$

which are in agreement with Planck+WP+BAO joint 95% CL contours for LFM ( $\mu \gtrsim 9M_{Pl}$ ) i.e.

$$\phi_0 \gtrsim 9M_{Pl}. \quad (116)$$

Although  $\phi_0$  is reduced compared to the original symmetry breaking case thanks to providing a longer curved trajectory with a smaller slope, the potential is still considered a super-Planckian model without any known physical motivation. Particularly, Picci-Quinn mechanism which is supposed to happen at QCD scale falls far below the needed energy of inflation and even the reheating process in the above hypothesis likely produces undesirable domain walls to explain the invisible axions.

The other probable scenario is rolling down the origin of field space. First, let us see which direction is more likely to be chosen for rolling down. Returning back to the field space polar coordinates we obtain

$$\frac{\partial V(R, \Theta)}{\partial \Theta} = 2\epsilon R^2 \sin \Theta \cos \Theta. \quad (117)$$

For a certain  $R$ , the  $\phi = 0$  line is always maximum while  $\psi = 0$  is a minimum so one probably expects slow-roll happen on the  $\phi$  axis or at least it appears as an effective attractor. But as it is also obvious from (117), near the origin all directions appear on the same footing so the field could follow different trajectories later on. In other words, the path chosen is very sensitive to the initial conditions and despite that the  $\phi$  axis is the most probable path, other options are not ruled out. Simulations confirm this idea (Figure 5). To analyze the inflationary scenario in this case, first let us derive  $\frac{d\psi}{d\phi}$  on the field space;

$$\frac{d\psi}{d\phi} = \frac{\psi(\phi^2 + \psi^2 - \phi_0^2 + \epsilon/\lambda)}{\phi(\phi^2 + \psi^2 - \phi_0^2)} = \frac{\psi}{\phi} \left( 1 - \frac{\epsilon/\lambda}{\phi_0^2 - \phi^2 - \psi^2} \right). \quad (118)$$

Even if the ratio  $\epsilon/\lambda\phi_0^2$  was not very small, we would rely on the smallness of  $\phi^2 + \psi^2$  with respect to  $\phi_0^2$  to establish the following approximation

$$\frac{d\psi}{d\phi} = \frac{\psi}{\phi} \left( 1 - \frac{\epsilon}{\lambda\phi_0^2} \right). \quad (119)$$

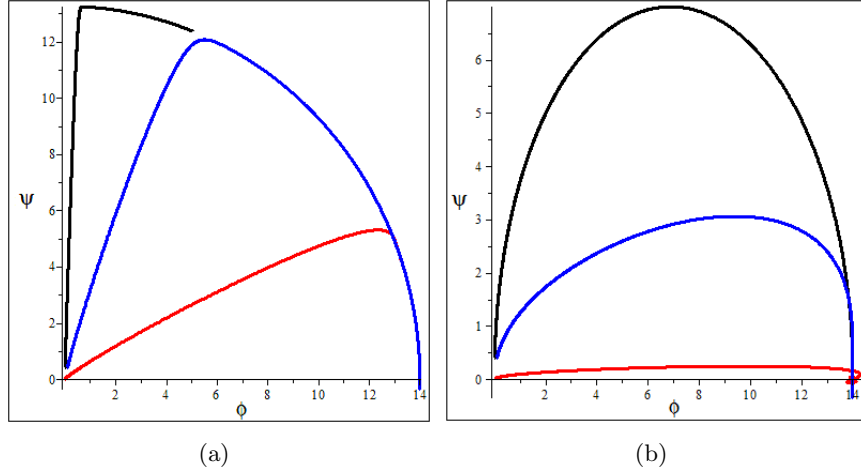


Figure 5: For  $\phi_0 \ll \frac{\epsilon}{\lambda}$ , the slow-roll path from the origin very well fits to a linear and a curved distinct parts (left), but as this inequality becomes weaker, the path exhibits more complexity (right), though still can be treated as the previous case with a weaker approximation.

The solution straightforwardly can be obtained as

$$\psi = A\phi^{1-\frac{\epsilon}{\lambda\phi_0^2}}, \quad (120)$$

in which  $A$  is an arbitrary constant which stems from arbitrariness in starting direction of the slow-roll. As  $\frac{\epsilon}{\lambda\phi_0^2}$  gets closer to 1, more sophisticated approximations will be necessary. For example, the above equation insures that the path outward origin remains linear with a good accuracy for small  $\frac{\epsilon}{\lambda\phi_0^2}$  ratio, until the path meets the radial minimum at  $\psi^2 + \phi^2 = \phi_0^2 - \epsilon/\lambda$  in which  $d\psi/d\phi$  quite suddenly vanishes. This abrupt redirection of course could raise the chance of isocurvature density perturbations [38, 75] for large  $\epsilon/\lambda\phi_0^2$  or even for small  $\epsilon/\lambda\phi_0^2$  ratio at the end of inflation. But remember that expansion appears as a damping term and prevents the field to acquire much kinetic energy and the subsequent tumbling. In this regard, numerical simulations confirm the slow-roll parameter predictions and slow-roll continues up to the true vacuum vicinity despite the slow-roll redirection in the field space (Figure 5(a)). For now, let us see how the potential looks in an arbitrary radial direction. To this end, we switch to the polar coordinates once more and write the potential for a fixed arbitrary angle  $\Theta_0$ .

$$V(R, \Theta = \Theta_0) = \frac{\lambda}{4}(R^2 - \phi_0^2)^2 + \frac{\epsilon}{2}R^2 \sin^2 \Theta_0 = \frac{\lambda\phi_0^4}{4} \left( 1 - \frac{2R^2}{\phi_0^2} \left( 1 - \frac{\epsilon}{\lambda\phi_0^2} \sin^2 \Theta_0 \right) + \frac{R^4}{\phi_0^4} \right). \quad (121)$$

Again, we encounter a hill-top model as could be expected and as mentioned before we shouldn't worry about a slow-roll interrupt in the redirection point since the inflation continues more or less in the same manner up to the true vacuum. Note that in a typical new inflation like what we consider, the first stages of inflation are much more effective in producing the e-foldings and the redirection always happens after these critical era such that gives us enough excuse for the mentioned approximations. This time we should define

$$\mu = \frac{\phi_0}{\sqrt{2 \left( 1 - \frac{\epsilon}{\lambda\phi_0^2} \sin^2 \Theta_0 \right)}}. \quad (122)$$

To check this against the previous results, we can take  $\epsilon = 0$  or  $\Theta_0 = 0$  expecting that both of these choices result in the constraint that we have already had from simple symmetry breaking in (67). Both of the above situations reduce (122) to

$$\mu = \frac{\phi_0}{\sqrt{2}}. \quad (123)$$

Then the hill-top constraint of  $\mu \gtrsim 9$  readily yields  $\phi_0 \gtrsim 12.72$  which is approximately in accord with the previously achieved constraint. On the other hand, the bigger  $\epsilon \sin^2 \Theta$ , the smaller  $\phi_0$  required for matching with the observation. Here a delicate point has to be taken into account. Although in the derivation of (122) we have not considered any constraint on the  $\frac{\epsilon}{\lambda\phi_0^2}$  ratio, but in order to have a multi-directional operation,  $\frac{\epsilon}{\lambda\phi_0^2} < 1$  is required, since the other case has the ability to bring about an imaginary  $\mu$  in some directions which consequently changes the sign of  $\mu^2$  in (122). Roughly speaking,  $\frac{\epsilon}{\lambda\phi_0^2}$  controls the range of the angle  $\omega$  in which the negative slope is seen from the origin: when  $\frac{\epsilon}{\lambda\phi_0^2} < 1$  then  $\omega = 2\pi$  but in the opposite case this angle for the positive half space ( $\phi > 0$ ) is obtained from

$$\frac{\omega}{2} = 2 \arcsin \sqrt{\frac{\lambda}{\epsilon}} \phi_0. \quad (124)$$

The division by two here results from considering half plane. This conclusion is not surprising at all since for  $\frac{\epsilon}{\lambda\phi_0^2} > 1$  the origin becomes a saddle point as discussed earlier. When the  $\frac{\epsilon}{\lambda\phi_0^2} > 1$  inequality gets stronger, the  $\phi$  axis plays the role of attractor more effectively. So in this regime we can ignore the  $\psi$  field in the slow-roll stage such that the potential reduces to

$$V(\phi, \psi = 0) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2, \quad (125)$$

which were discussed fully earlier. It might be worth arguing that in the last case (i.e.  $\frac{\epsilon}{\lambda\phi_0^2} > 1$ ), the potential for the initial condition in which fields are located far from origin, imitates the chaotic inflation in the form of  $V(\psi) = \frac{\epsilon}{2}\psi^2$  which continues along the trajectory to the true vacuum.

In Figure 6(a) one observes the curved path from the saddle point on the  $\psi$  axis toward the true vacuum at  $(\phi = \phi_0, \psi = 0)$ . To plot these graphs we simply calculated the absolute slope by

$$\vec{\nabla} V(\phi, \psi) = \frac{\partial V}{\partial \phi} \hat{\phi} + \frac{\partial V}{\partial \psi} \hat{\psi}. \quad (126)$$

To be more rigorous, the length of the projected interval between the two saddle points on the  $\phi - \psi$  plane is  $2\sqrt{\phi_0^2 - \epsilon/\lambda}$  and disappears as soon as  $\epsilon/\lambda$  reaches  $\phi_0^2$  (or gets bigger). Therefore, it is natural to take into account the new path provided for the fields slow-roll. To indicate what we exactly talk about one can refer to Figure 7(a). As it is depicted in the figures, in the proposed potential, the slow-roll path could be completely different from the radial flow of an ordinary symmetry breaking. It is saying that the path is very sensitive to the initial conditions and the figures are just two possible paths among many.

Note that in Figure 7(a) the slow-roll comprises of two different paths; a radial and a nearly circular path, in contrast to the  $\lambda\phi_0^2 \ll \epsilon$  case, in which the saddle points unite at the origin. As we have shortly discussed under the motivation topic, the latter case approximately reduces to the inverted hybrid inflation case [60], with high energy scale domain walls which are formed before the inflationary procedure. Let us get back to the main trend and suppose  $\lambda\phi_0^2 > \epsilon$ . It

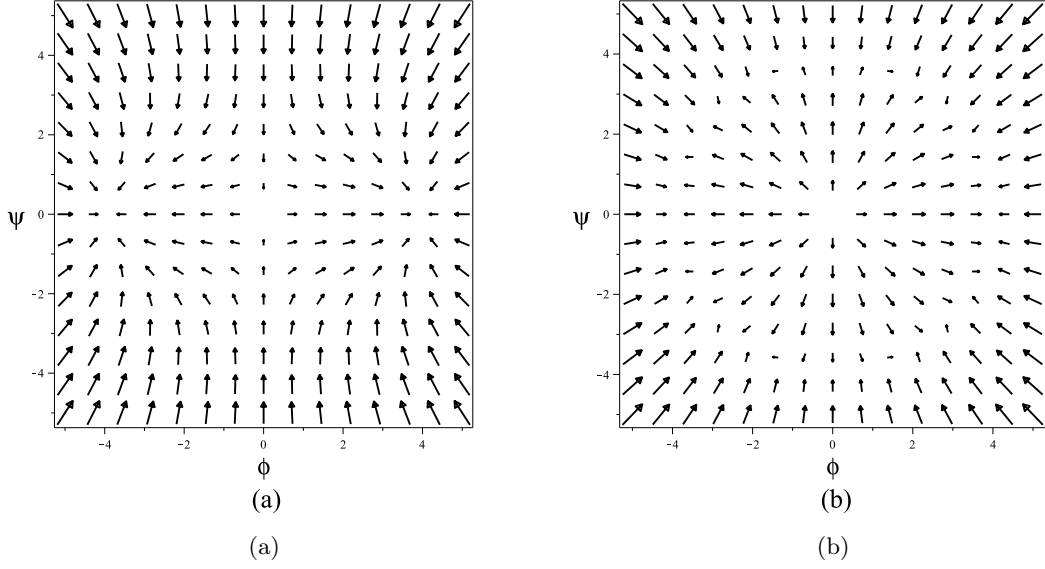


Figure 6: The above arrow diagrams show the direction of the gradient vector field of the potential in the  $\phi$ - $\psi$  field coordinates. In order to clarify the demonstration the gradient vector field has been plotted in logarithmic scale. The diagram on the left is plotted for  $\lambda\phi_0^2 < \epsilon$ , while the right one is for  $\lambda\phi_0^2 > \epsilon$ . It is obvious that the saddle points unite at the origin for the first choice of the parameters.

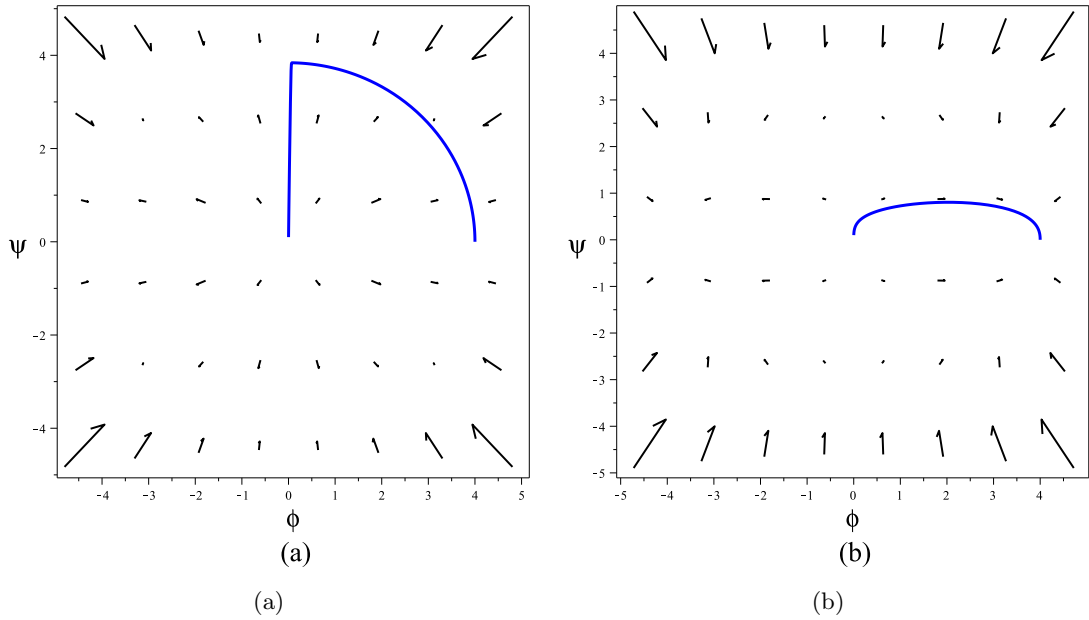


Figure 7: The above graphs shows a sample line flow versus gradient vectors. The left graph corresponds to the case  $\lambda\phi_0^2 \gg \epsilon$  in which we have a central maximum and two side saddle point. The right graph respectively correspond to the case where  $\lambda\phi_0^2 \simeq \epsilon$  where the origin turns to become a saddle point.

was discussed earlier that the path consists of two distinct parts, a radial and a nearly circular one, although the circular part changes in shape as the inequality of  $\lambda\phi_0^2 > \epsilon$  becomes weaker. One has to note that if the slow-roll happens to be on the  $\phi$  axis, then again the problem reduces to the ordinary symmetry breaking scenario and the circular path doesn't appear anymore. The remaining possibility that has less importance relates to the case when  $\phi_0^2$  is comparable to  $\epsilon/\lambda$ . As it was discussed earlier, in this case the slow-roll path is a curved line (Figure 7(b)) and for such trajectories, there are other methods to deal with the slow-roll process [76]-[79]. However, since recent observations are generally in favor of single field inflation, or at last, such models that evolve along an effectively single-field attractor solution [80], we limited our survey to the situations in which we can approximate our double field potential with a single field one.

## 6 Conclusion

We estimated the domain wall properties for an explicitly broken  $U(1)$  symmetric potential introducing an approximation that nicely fits both the Euler-Lagrange equations with appropriate boundary conditions and the static version of viral theorem. We showed that adding one degree of freedom into our Lagrangian in the form of a new field, helps us evade the domain wall domination problem of the ordinary  $\phi^4$  kink without decreasing the scale energy of the potential. The price that has to be paid is relaxing the  $U(1)$  symmetry as an exact symmetry of the model. This allows us to have super-Planckian scale of energy for the peak of the potential while the domain wall energy is sufficiently low to avoid conflict with observation. The exact allowed values of parameters are of course model dependent. More rigorously, it has to be first determined, in which cosmological era and correlation length (horizon), the domain walls will form. The descending of  $U(1)$  to  $Z_2$  is not an unprecedented scenario and the same explicit symmetry breaking have been suggested as a remedy for invisible axion and two Higgs doublet models. From an observational point of view, the model parameters could be set such that the wall explains any confirmed CMB residual dipole anisotropy. In the other extreme, we proposed the domain wall production to happen before inflation and claimed that for super-Planckian values, this scenario could work properly.

We thoroughly examined the potential as the source of inflation. We just focused on those cases which are reduced to a single field inflation since such models have received more appreciation after the Planck data. Our study indicated that all the mentioned scenarios could be classified into the "new inflationary models" and almost always into the hill-top subclass of it. We also introduced an analytic, though approximate proof for the well-known simple symmetry breaking potential which indicates nearly complete accordance with the previously obtained numerical values. We tried to encompass all inflationary possibilities of the potential. As briefly mentioned, there is hope to explain the possible CMB dipole anisotropy by means of domain walls, which could simultaneously solve the invisible axion problem. Therefore, it is important to make a compromise between cosmological evidence, particle physics requirements and domain wall formation as we tried to do so.

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